Gravitation

- **Kepler's law of planetary motion:**

① All planets move in elliptical orbits with the sun at a focus.

② The radius vector from the sun to the planet sweeps equal area in equal time.

③ The square of the time period of a planet is proportional to the cube of the semimajor axis of the ellipse.

- Newton found that the acceleration of a body towards the earth is inversely proportional to the square of the distance of the body from the centre of earth.

Thus, $a \propto \frac{1}{r^2}$

Also, the force is mass times acceleration and so it is proportional to the mass of the body. Hence,

$$F \propto \frac{m}{r^2}$$

By the third law of motion, the force on a body due to the earth must be equal to the force on the earth due to the body. Therefore, this force should also be proportional to the mass of the earth. Thus, the force between the earth and a body is,

$$F \propto \frac{Mm}{r^2}$$

or, $$\boxed{F = \frac{GMm}{r^2}} \quad \text{--- (1)}$$

Newton further generalized the law by saying that not only the earth but all material body in the universe attract each other according to eq^n (1), with same value of G.

$G \rightarrow$ Universal constant of gravity

$$= 6.67 \times 10^{-11}\ N\text{-}m^2/kg^2$$

Eq^n (1) is called the Universal law of gravitation.

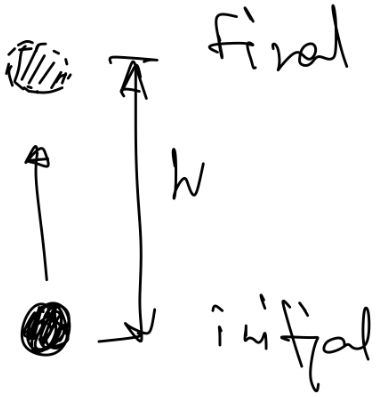
– The first important fundamental measurement of the quantity (G) was made by Cavendish in 1736 about 71 years after the law was formulated.

III Gravitational Potential Energy:

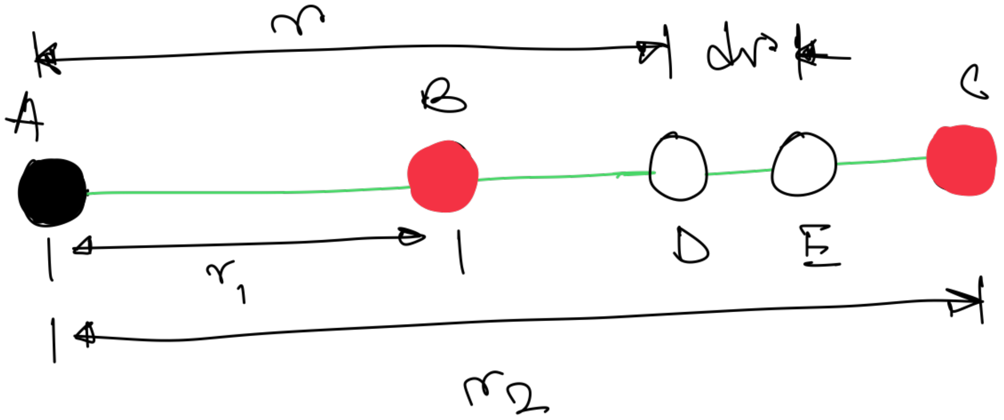
The potential energy of a system corresponding to a conservative force was defined as,

$$U_f - U_i = -\int_i^f \vec{F}\cdot d\vec{r}$$

The change in potential energy is equal to the negative of the work done by the internal forces.



$$U_f - U_i = mgh$$



$m_1 \qquad m_2 \dashrightarrow m_2$

Let two particles are at A & B, the masses are m_1 & m_2 respectively. Now, m_2 is moved from B to C.

Initially, $AB = r_1$ & finally $AC = r_2$.

We have to calculate the change in potential of the system of the two particles as the distance changes from r_1 to r_2.

Consider a small displacement when the distance b/w the particles changes from r to $r + dr$ i.e from D to E.

The force of the second particle i.e. m_2,

$$F = \frac{G m_1 m_2}{r^2} \text{ along } \overrightarrow{DA}$$

The work done by the gravitational force in the displacement is,

$$dW = -\frac{G m_1 m_2}{r^2} \cdot dr$$

The increase in potential energy of the two-particle system during this displacement is,

$$dU = -dW = \frac{G m_1 m_2}{r^2} dr.$$

The increase in potential energy as the distance between the particles changes from r_1 to r_2 is—

$$U(r_2) - U(r_1) = \int dU$$

$$= \int_{r_1}^{r_2} \frac{G m_1 m_2}{r^2} \cdot dr$$

$$= \frac{G m_1 m_2}{r2} \left[\frac{1}{r_1} - \frac{1}{r_2} \right] \quad \text{———} (2)$$

From eqn (2), if the seperation b/w two particle system is infinite

$U(r') = 0$.

When the seperation is from r' to r the potential energy,

$$U(r) - U(r') = Gm_1m_2\left[\frac{1}{r'} - \frac{1}{r}\right]$$

$$= -\frac{Gm_1m_2}{r}$$

* If there are three particles A, B and C, there are three pairs AB, AC and BC. The potential energy of the three-particle system is equal to the sum of the potential energies of the three pairs.

For an N particle system there are $\frac{N(N-1)}{2}$ pairs. And potential energy calculated for each pair and added to get the total potential energy of the system.

III Gravitational Potential

Suppose a particle of mass m is taken from a point A to a point B. Let U_A & U_B denote the gravitational potential energy when the mass m is at point A & B respectively.

We define the 'change in potential' $V_B - V_A$ between the two points as

$$V_B - V_A = \frac{U_B - U_A}{m}$$

We choose any point to have zero potential. Such a point is called a reference point. If A is the reference point, $V_A = 0$, and

$$V_B = \frac{U_B - U_A}{m} \qquad (3)$$

* Thus, gravitational potential at a point is equal to the change in potential energy per unit mass, as the mass is brought from the reference point to the given point.

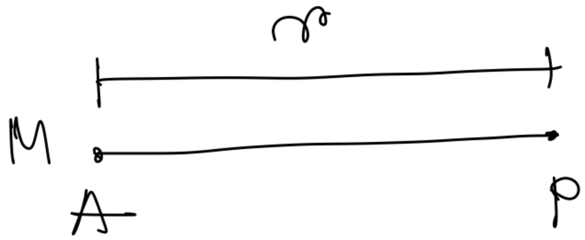
The potential at a point may also be defined as the work done per unit mass by an external agent in bringing a particle slowly from the reference point to the given point.

Generally the reference point is chosen at infinity so that the potential at infinity is zero.

SI unit of gravitational potential is $J\,kg^{-1}$.

(A) Potential due to a Point Mass

Suppose a particle of mass M is kept at a point A and we have to calculate the potential at a point P at a distance r away from A. The reference point is at Infinity.



So, $V(r) = \dfrac{U(r) - U(\infty)}{m}$ from eqn (3)

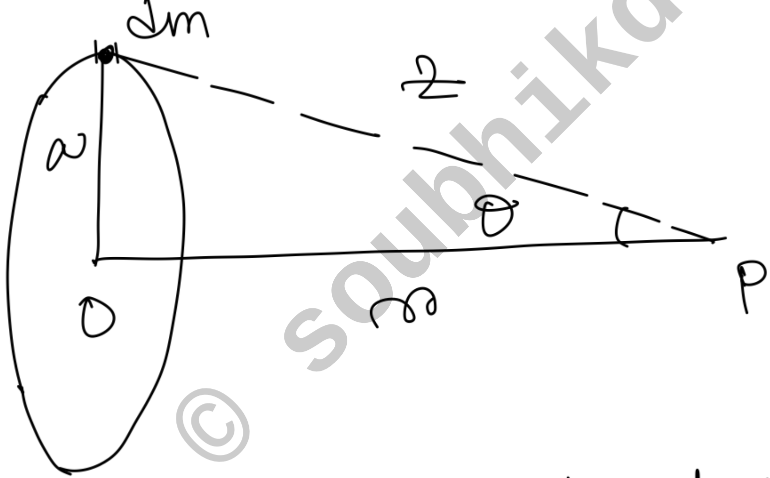
But, $U(r) - U(\infty) = -\frac{GMm}{r}$

So, $V = -\frac{GM}{r}$

The gravitational potential due to a point mass M at a distance r is,

$$-\frac{GM}{r}$$

(B) **Potential due to a uniform ring at a point on its axis**



Let the mass of the ring is M and radius a.

Calculate gravitational potential at a point P on the axis of the ring.

$OP = r$

Consider any small part of the ring of mass dm.

So, $z = \sqrt{a^2 + r^2}$

Potential at P due to dm is —

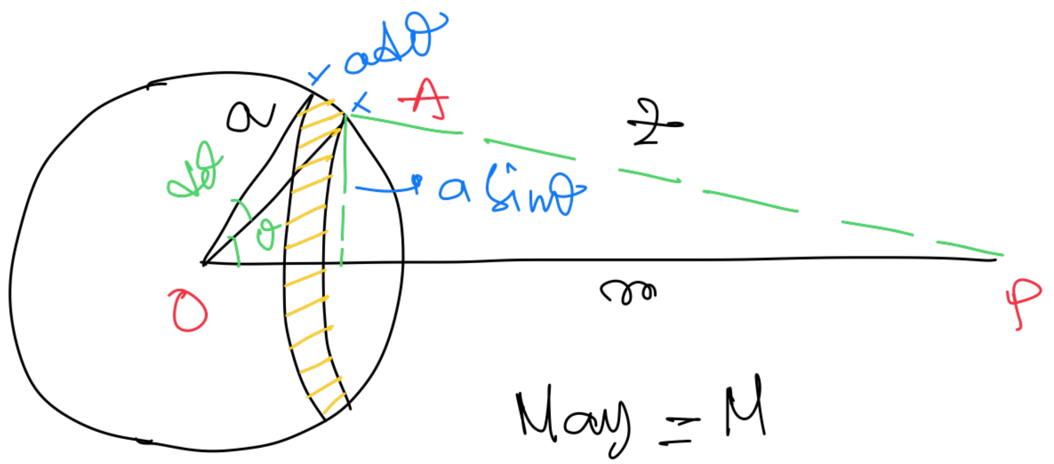
$$dV = -\frac{G\,dm}{z} = -\frac{G\,dm}{\sqrt{a^2+r^2}}$$

The potential of V due to the whole ring of mass M,

$$V = \int dV = \int -\frac{G\,dm}{\sqrt{a^2+r^2}}$$

$$V = -\frac{GM}{\sqrt{a^2+r^2}}$$

(c) $\boxed{\text{Potential due to a uniform thin spherical shell.}}$



Mass = M

Radius = a

Calculate V at P due to the shell.

Mass per unit area $= \dfrac{M}{4\pi a^2}$

The radius of the ring (yellow) is $a\sin\theta$. Hence the perimeter is $2\pi a\sin\theta$.

The width of the ring is $a\,d\theta$.

The area of the ring is —

$$2\pi a\sin\theta \cdot a\,d\theta = 2\pi a^2\sin\theta\,d\theta.$$

As the shell is uniform, the mass of the ring is,

$$\frac{M}{4\pi a^2}\left(2\pi a^2\sin\theta\,d\theta\right)$$

$$= \frac{M}{2}\sin\theta\,d\theta$$

From the triangle OAP, if $AP = z$, $OP = r$

$$z^2 = (a\sin\theta)^2 + (r - a\cos\theta)^2$$

$$= a^2\sin^2\theta + r^2 + a^2\cos^2\theta - 2ar\cos\theta$$

$$z^2 = a^2 + r^2 - 2ar\cos\theta \quad \text{———} \quad (i)$$

Taking derivatives of eq^n (i)

$$2z \cdot dz = 2ar \sin\theta \, d\theta$$

$$\text{or,} \quad \sin\theta \, d\theta = \frac{2 \, dz}{ar}$$

Thus the mass of the ring is –

$$dm = \frac{M}{2ar} \, 2 \, dz$$

As the distance of any point of the ring from P is z, the potential at P due to the ring is –

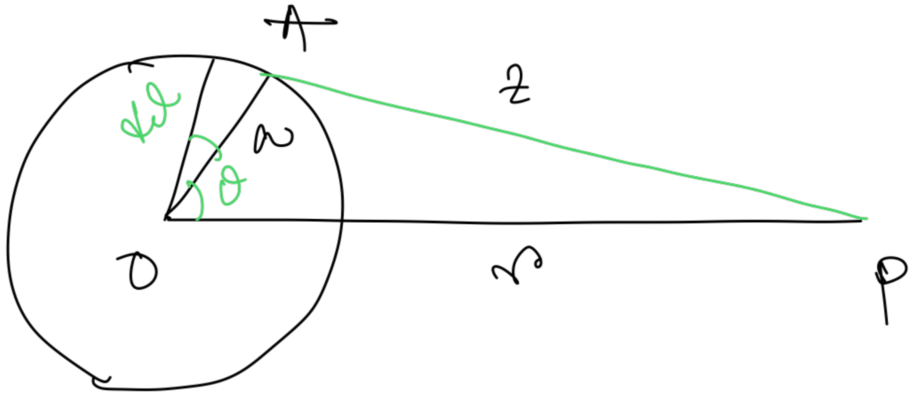
$$dV = -\frac{G \, dm}{z}$$

$$dV = -\frac{GM}{2ar} \, dz$$

So, V for the whole shell,

$$\boxed{V = \int_0^{\pi} -\frac{GM}{2ar} \, dz} \quad \text{—— (4)}$$

Case I; P is outside the shell ($r > a$)



when $\theta = 0$; $z = r - a$

when $\theta = \pi$; $z = r + a$

Thus as θ varies from 0 to π, z varies from $(r-a)$ to $(r+a)$.

Thus,

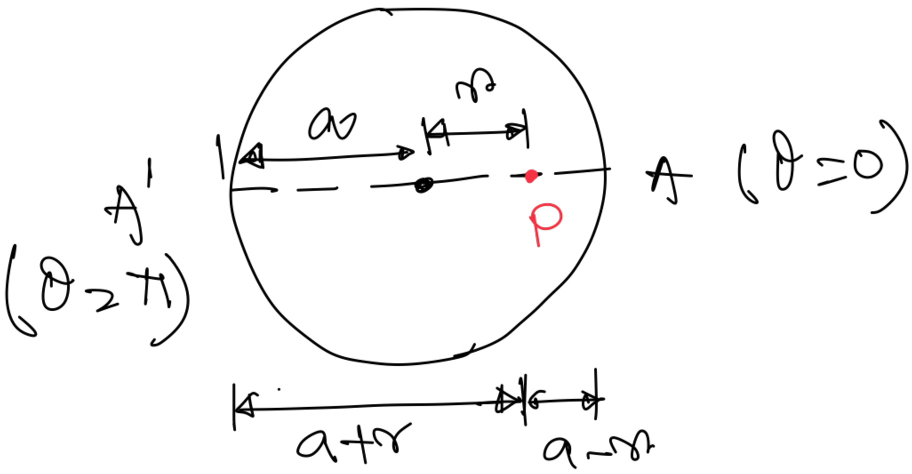
$$V = \int dV = \int_{r-a}^{r+a} -\frac{GM}{2ar}\, dz$$

$$= -\frac{GM}{2ar}\left[z\right]_{r-a}^{r+a}$$

$$= -\frac{GM}{r}$$

* To calculate the potential at an external point, a uniform spherical shell may be treated as a point particle of equal mass placed at its centre.

Case 2 : P is inside the shell ($r < a$)



when $\theta = 0$; $z = a - r$

when $\theta = \pi$; $z = a + r$

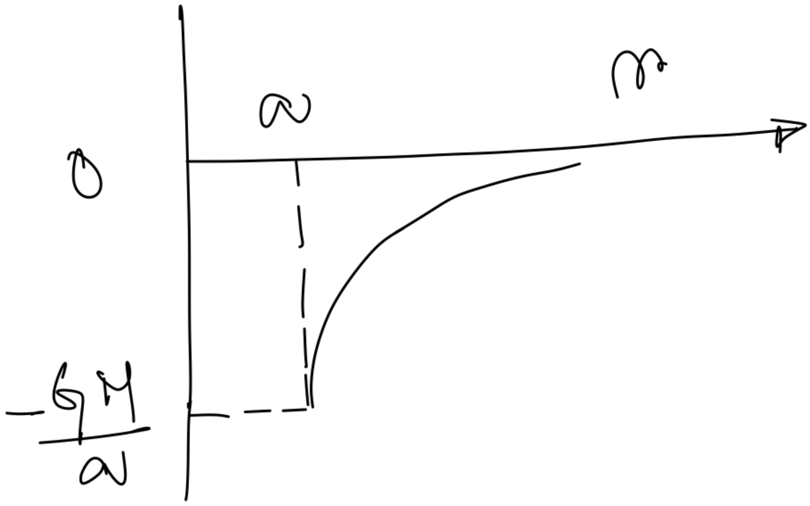
So, for θ, 0 to π, z varies from $(a-r)$ to $(a+r)$.

Thus, $$V = -\frac{GM}{2ar}\int_{a-r}^{a+r} dz$$

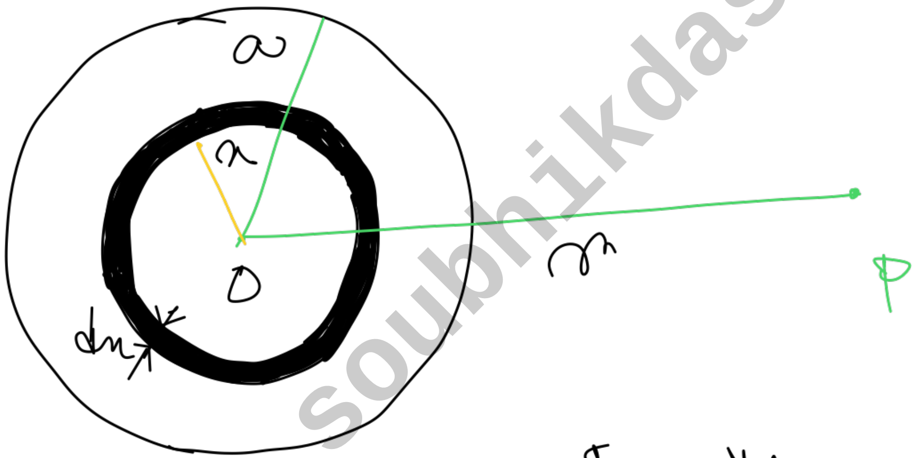
$$= -\frac{GM}{2ar}\left[z\right]_{a-r}^{a+r}$$

$$= -\frac{GM}{a}$$

* This doesn't depend on r. Thus the potential is constant throughout the cavity of the shell.



(D) Potential due to a uniform solid sphere.



Mass of the sphere is M. Radius $= a$. Calculate gravitational potential at point P. Let $OP = r$.

$$\text{Mass per unit volume} = \frac{M}{\frac{4}{3}\pi a^3}$$

Volume of the spherical shell

$$= 4\pi x^2 \cdot dx$$

So, mass of the spherical shell,

$$dm = \frac{M}{\frac{4}{3}\pi a^3} \cdot 4\pi x^2 \cdot dx$$

$$= \frac{3M}{a^3} x^2 \cdot dx$$

The potential due to the shell at point P,

$$dV = -\frac{G\,dm}{r} \text{ if } x < r$$

$$= -\frac{G\,dm}{x} \text{ if } x > r$$

$\boxed{\text{Case 1: Potential at an external point}}$

(r > a)

$$dV = -\frac{G\,dm}{r}$$

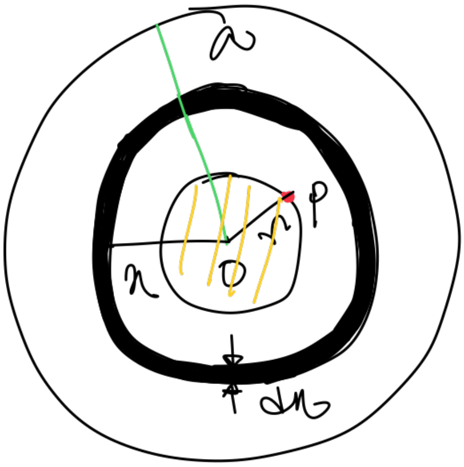
$$V = \int dV = -\frac{G}{r}\int dm$$

$$= -\frac{GM}{r}$$

* Potential is same as that due to a single particle of equal mass placed at its centre.

Case 2: Potential at an internal point

($x < r$)



Divide the sphere in two parts by imagining a concentric spherical surface passing through P.

The inner part has a mass,

$$M' = \frac{M}{\frac{4}{3}\pi a^3} \times \frac{4}{3}\pi r^3$$

$$= \frac{M r^3}{a^3}$$

The potential at P due to this inner part

$$V_1 = -\frac{G M'}{r}$$

$$= -\frac{G M r^2}{a^3}$$

To get the potential at P due to the outer part of the sphere, we divide this part in concentric shells.

The mass of the shell b/w radii x & $x+dx$ is —

$$dm = \frac{M}{\frac{4}{3}\pi a^3} \cdot 4\pi x^2 \cdot dx$$

$$= \frac{3 M x^2 dx}{a^3}$$

The potential at point P due to the shell,

$$-\frac{G\,dm}{x} = -\frac{3GM}{a^3} x\,dx$$

Thus, the potential due to the outer part is,

$$V_2 = \int_r^a -\frac{3GM}{a^3} x\,dx$$

$$= -\frac{3GM}{a^3}\left[\frac{x^2}{2}\right]_r^a$$

$$= -\frac{3GM}{a^3}\left(a^2 - r^2\right)$$

Total potential at P is, $V = V_1 + V_2$

$$V = -\frac{GMr^2}{a^3} - \frac{3GM}{a^3}(a^2 - r^2)$$

$$= -\frac{GM}{2a^3}(3a^2 - r^2)$$

* At the centre of the sphere the $(r=0)$ potential is,

$$\boxed{V = -\frac{3GM}{2a}}$$

III Gravitational field

→ A body creates a gravitational field in the space around it. The field has its own existence and has energy and momentum. This field has a definite direction at each point of the space and its intensity varies from point to point.

The direction and intensity of the field is defined in terms of the force it exerts on a body placed in it. We define the intensity of gravitational field $\vec{E}$ at a point by, $\boxed{\vec{E} = \frac{\vec{F}}{m}}$

where $\vec{F}$ is the force exerted by the field on a body of mass m.

SI unit is $N\,kg^{-1}$.

* Gravitational field adds according to the rules of vector addition.

$$\vec{E} = \vec{E_1} + \vec{E_2}$$

III Relation between Gravitational field and potential.

— Suppose the gravitational field at a point $\vec{r}$ due to a given mass distribution is $\vec{E}$.

The force on a particle of mass m, when it is at $\vec{r}$ is —

$$\vec{F} = m\vec{E}$$

As the particle is displaced from $\vec{r}$ to $\vec{r} + \vec{dr}$ the work done by the gravitational force on it is,

$$dW = \vec{F} \cdot \vec{dr}$$

$$= m\vec{E} \cdot \vec{dr}$$

The change in potential energy during this displacement is,

$$dU = -dW = -m\vec{E}\cdot\vec{dr}$$

So, $$dV = \frac{dU}{m} = -\vec{E}\cdot\vec{dr}$$

Integrating between r_1 & r_2,

$$\vec{V}(r_2) - \vec{V}(r_1) = -\int_{r_1}^{r_2}\vec{E}\cdot\vec{dr}$$

If r_1 is taken as the reference point, $\vec{V}(r_1) = 0$. The potential $\vec{V}(r)$ at any point $\vec{r}$ is,

$$\vec{V}(r) = -\int_{\vec{r_0}}^{\vec{r}}\vec{E}\cdot\vec{dr}$$

$\vec{r_0}$ denotes the reference point.

* For cartesian coordinates,

$$\vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{K}$$

and $$\vec{dr} = dx\vec{i} + dy\vec{j} + dz\vec{K}$$

so that, $$\vec{E}\cdot\vec{dr} = E_x dx + E_y dy + E_z dz$$

As, $dV = -\vec{E}\cdot\vec{dr}$

$$dV = -E_x dx - E_y dy - E_z dz$$

If y and z remain constant, $dy = dz = 0$

Thus, $E_x = -\dfrac{\delta V}{\delta x}$

Similarly, $E_y = -\dfrac{\delta V}{\delta y}$

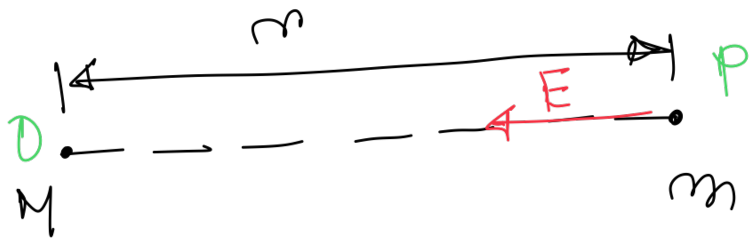
$$E_z = -\frac{\delta V}{\delta z}$$

$\dfrac{\delta}{\delta x}$ means partial differention w.r.t x treating y and z to be constants.

* The field may be obtained by differentiating the potential. $\left(E_x = -\dfrac{\delta V}{\delta x}\right)$

III Calculation of Gravitational field

Ⓐ $\boxed{\text{Field due to a point mass}}$



Particle of masses M and m are placed at O and P respectively. Let $OP = r$.

The mass M creates a field $\vec{E}$ at the site of mass m and this field exerts a force $\vec{F} = m\vec{E}$ on the mass m.

But the force $\vec{F}$ on the mass m due to the mass M is,

$$F = \frac{GMm}{r^2} \text{ acting along } \vec{PO}.$$

Then the gravitational field at P is,

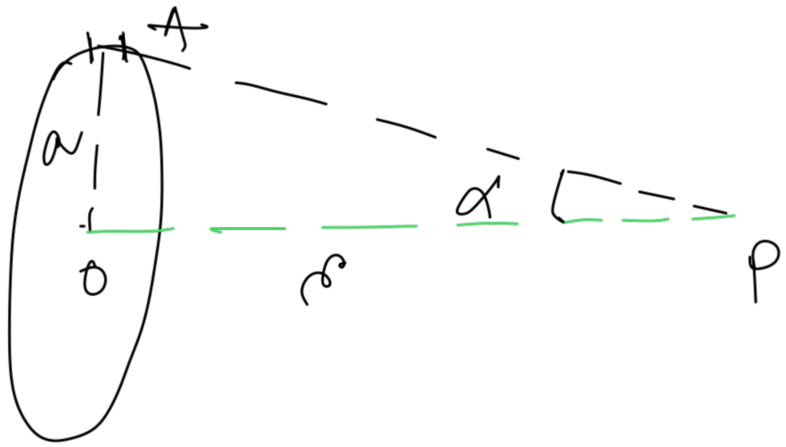
$$E = \frac{GM}{r^2} \text{ along } \vec{PO}.$$

If O is taken as the origin, the position vector of mass m is $\vec{r} = \vec{OP}$

If $\vec{e_r}$ is the unit vector along $\vec{r}$,

$$\boxed{\vec{E} = -\frac{GM}{r^2}\vec{e_r}}$$

(3) Field due to a uniform circular ring at a point on its axis



Mass $= M$

Radius $= a$

calculate gravitational field at P at a distance r from the centre.

- The field must be towards centre $\vec{PO}$.

Considering any particle of mass dm on the ring, say at point A.

Now, $AP = z = \sqrt{a^2 + r^2}$.

The gravitational field at P due to dm is along $\vec{PA}$ and its magnitude is

$$dE = \frac{G\,dm}{z^2}$$

the component along PO is

$$dE\cos\alpha = \frac{G\,dm}{z^2}\cos\alpha$$

The net gravitational field at P due to the ring is –

$$E = \int \frac{G\,dm}{z^2} \cos\alpha = \frac{GM}{z^2} \cos\alpha$$

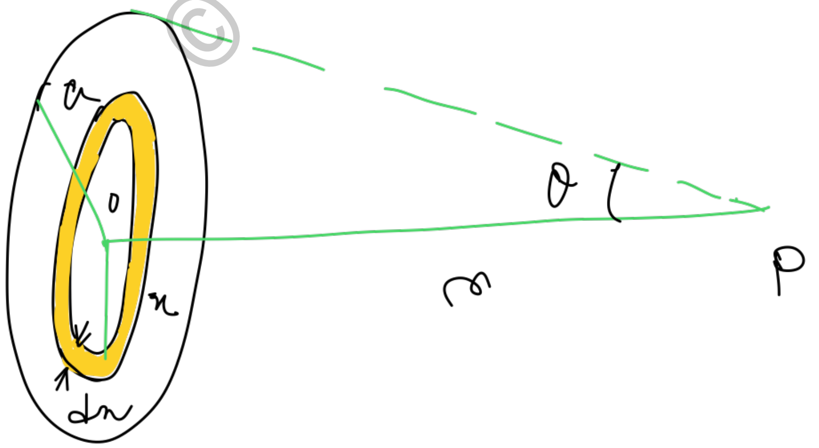
$$\boxed{E = \frac{GMr}{(a^2+r^2)^{3/2}}}$$

$$z = \sqrt{a^2+r^2}$$

$$\cos\alpha = \frac{r}{\sqrt{a^2+r^2}}$$

The field is directed towards the centre of the ring.

(c) Field due to a uniform disc at a point on its axis



Mass = M

Radius = a

Distance of the point P on its axis from the centre is r.

Find gravitational field at P due to the disc.

– Let us draw a circle of radius x with the centre at O. Draw another concentric circle of radius $x+dx$. The part of the disc enclosed between these two circles can be treated as a uniform ring of radius x.

The area of the ring, $2\pi x dx$.

The mass of the ring,

$$dm = \frac{M}{\pi a^2} \cdot 2\pi x \, dx$$

$$= \frac{2Mx\,dx}{a^2}$$

The gravitational field at P due to the ring is,

$$dE = \frac{G\left(\frac{2Mx\,dx}{a^2}\right)r}{(r^2+x^2)^{3/2}}$$

$$= \frac{2GMr}{a^2} \cdot \frac{x\,dx}{(r^2+x^2)^{3/2}}$$

As x varies from 0 to a, the net field,

$$E = \int_0^a \frac{2GMr}{a^2} \cdot \frac{x\,dx}{(r^2+x^2)^{3/2}}$$

$$= \frac{2GMr}{a^2} \int_0^a \frac{x\,dx}{(r^2+x^2)^{3/2}}$$

Let, $r^2 + x^2 = z^2$

Now, $2x\,dx = 2z\,dz$

and $\displaystyle\int \frac{x\,dx}{(r^2+x^2)^{3/2}} = \int \frac{z\,dz}{z^3}$

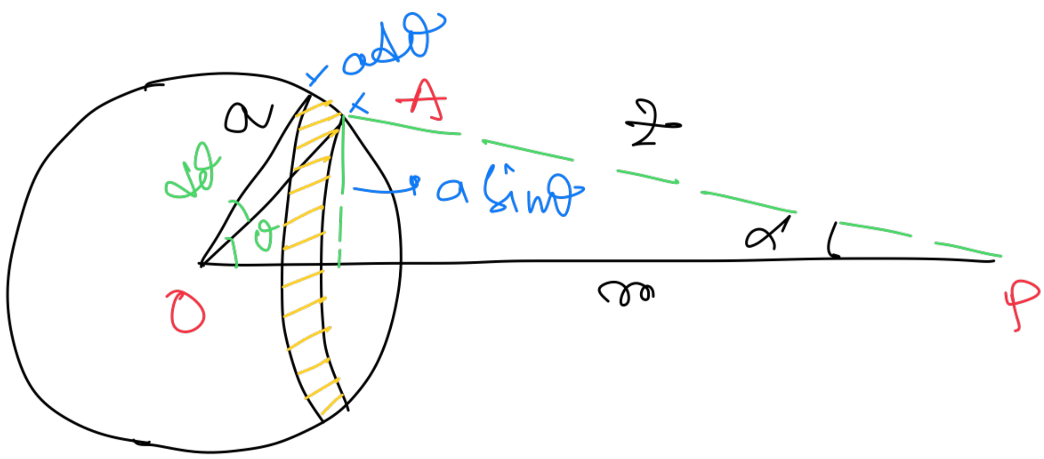
$$= -\frac{1}{z} = -\frac{1}{\sqrt{r^2+x^2}}$$

So, $$E = \frac{2GMr}{a^2}\left[-\frac{1}{\sqrt{r^2+x^2}}\right]_0^a$$

$$= \frac{2GMr}{a^2}\left[\frac{1}{r} - \frac{1}{\sqrt{r^2+a^2}}\right]$$

$$E = \frac{2GM}{a^2}(1-\cos\theta)$$

(D) $\boxed{\text{Field due to a uniform thin spherical shell.}}$



Mass $= M$

Radius $= a$

The mass of the shaded ring,

$$dm = \frac{M}{2}\sin\theta\, d\theta$$

The field at P due to the ring,

$$dE = \frac{G\,dm}{z^2}\cos\alpha = \frac{GM}{2}\cdot\frac{\sin\theta\, d\theta\cos\alpha}{z^2}$$

from OAP,

$$z^2 = a^2 + r^2 - 2ar\cos\theta$$

$$\text{or, } \sin\theta\, d\theta = \frac{2z\,dz}{ar}$$

$$a^2 = z^2 + r^2 - 2zr\cos\alpha$$

$$\text{or, } \cos\alpha = \frac{z^2 + r^2 - a^2}{2zr}$$

so, $E = \int dE = \frac{GM}{4ar^2}\left[z + \frac{a^2-r^2}{z}\right]$

Case 1: P is outside the shell ($r>a$)

— z varies from $r-a$ to $r+a$.

The field,

$$E = \frac{GM}{4ar^2}\left[z + \frac{a^2-r^2}{z}\right]_{r-a}^{r+a}$$

$$E = \frac{GM}{r^2}$$

Case 2: P is inside the shell ($r<a$)

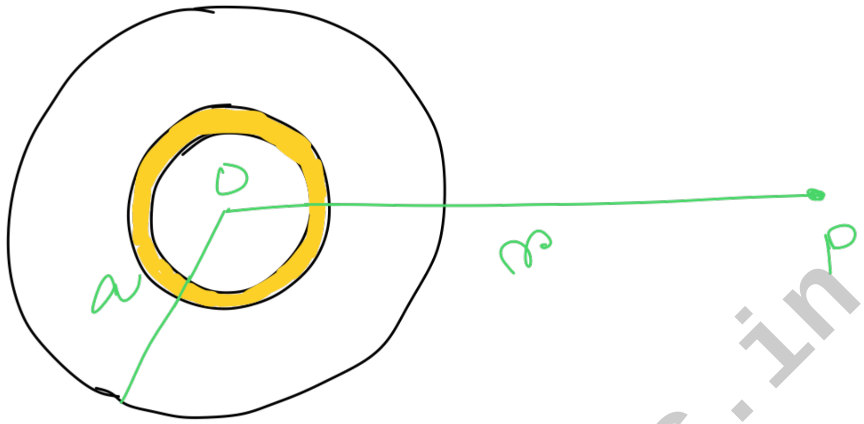
— z varies from $a-r$ to $a+r$.

$$E = 0$$

* Hence the field inside a uniform spherical shell is zero.

(E) $\boxed{\text{Gravitational field due to a uniform solid sphere.}}$

Case 1 : field at an external point ($r > a$)



Mass is M and Radius a.

Find gravitational field due to a point P outside of the sphere at a distance r.

— Divide the sphere into thin spherical shells each centred at O.

Let the mass of one such shell is dm.

The field at P due to the shell,

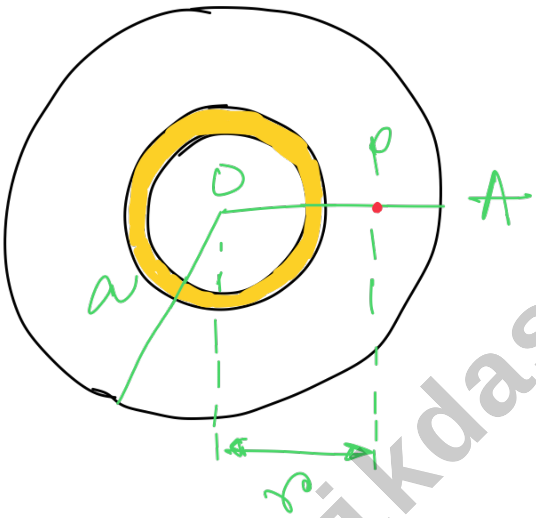
$$dE = \frac{G\,dm}{r^2} \text{ toward } \vec{PO}.$$

So, field due to the whole sphere —

$$E = \int dE = \frac{G}{r^2}\int dm = \frac{GM}{r^2}$$

Thus a uniform sphere may be treated as a single particle of equal mass placed at its centre for calculating the gravitational field at an external point.

Case 2: Field at an internal point. $(r<a)$



Mass $= M$

Radius $= a$

– let the mass of the shell is dm.

The field due to the shell,

$$E = \frac{G\,dm}{r^2} \text{ along } \vec{PO}.$$

$$E = \int dE = \frac{G}{r^2}\int dm.$$

only the masses of the shells with radii less than r should be added to get $\int dm$.

The mass of the sphere of radius r,

$$\frac{M}{\frac{4}{3}\pi a^3} \cdot \left(\frac{4}{3}\pi r^3\right) = \frac{Mr^3}{a^3}$$

Then, $\int dm = \frac{Mr^3}{a^3}$

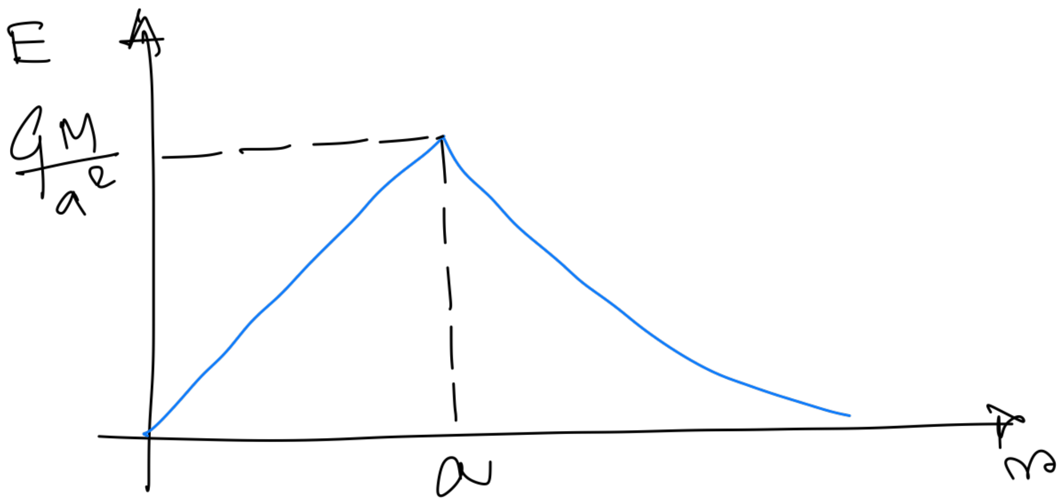
So, $E = \frac{GM}{r^2 \cdot a^3} r^3$

$$\boxed{E = \frac{GM}{a^3} r}$$

If $r = 0$; $E = 0$

$r = a$; $E = \frac{GM}{a^2}$

Any particle at the centre is equally pulled from all sides and the resultant must be zero.



[VI] Variation in the value of 'g'

The acceleration due to gravity is given by,
$$g = \frac{F}{m}$$

(a) Height from the surface of the Earth.

→ If the object is placed at a distance h above the surface of the earth, the force of gravitation on it due to the earth is,

$$F = \frac{GMm}{(R+h)^2}$$

M and R are the mass and radius of the earth.

Thus,
$$g = \frac{F}{m} = \frac{GM}{(R+h)^2} = \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2}$$

$$= \frac{g_0}{\left(1+\frac{h}{R}\right)^2}$$

Where, $g_0 = \frac{GM}{R^2}$ is the value of g at the surface of the earth.

If $h << R$

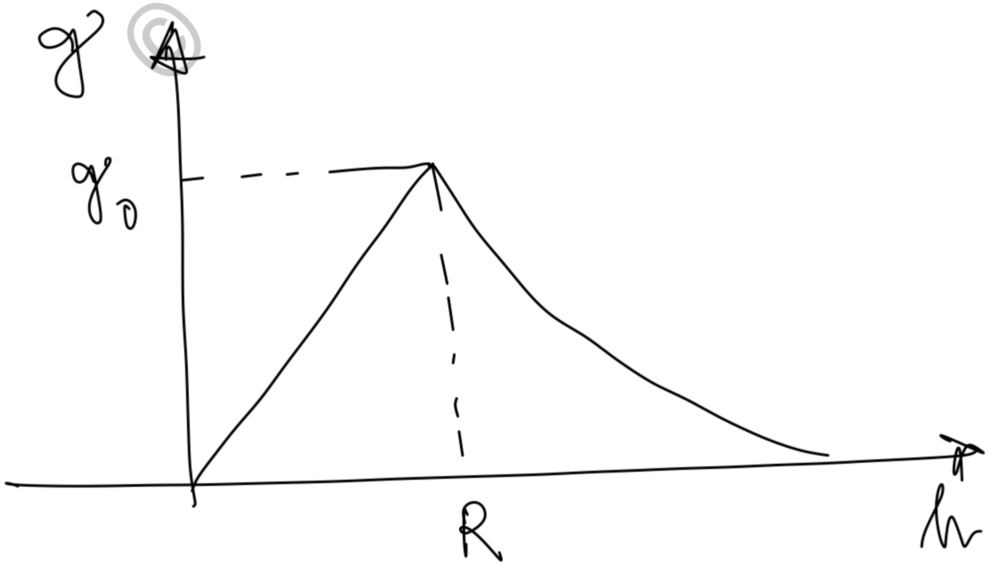
$$g = g_0\left(1+\frac{h}{R}\right)^{-2} \simeq g_0\left(1-\frac{2h}{R}\right)$$

If one goes a distance h inside the earth such as in mines, the value of g decreases. The force by earth is (case 2: field at an internal point)

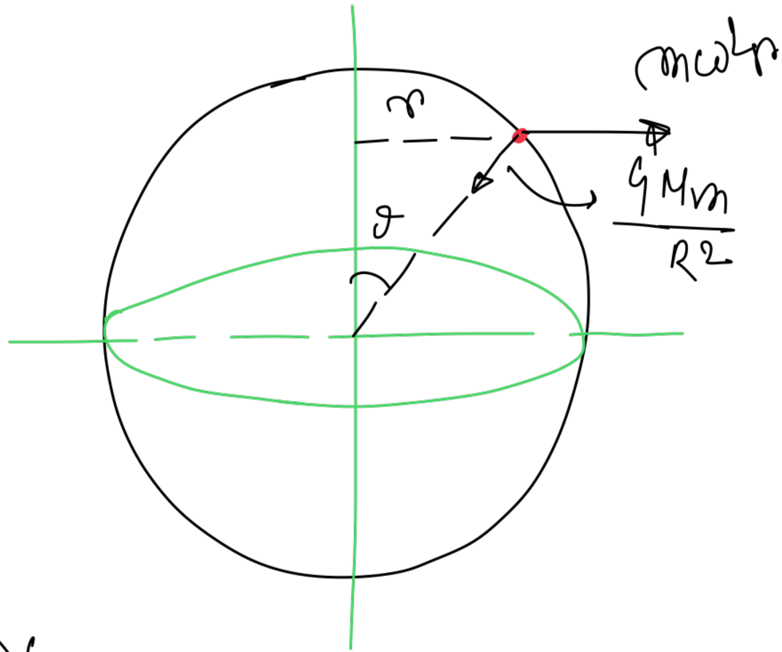
$$F = \frac{GMm}{R^3}(R-h)$$

$$g = \frac{F}{m} = \frac{GM}{R^3}(R-h)$$

$$= g_0\left(1-\frac{h}{R}\right)$$



(b) Rotation of the earth.



- As the earth rotates about its own axis the frame attached to the earth is noninertial.

To use Newton's laws, we have to include pseudo forces.

For an object at rest with respect to the earth, a centrifugal force $m\omega^2 r$ is to be added. Where m is the mass, ω is the angular velocity of the earth and r is the radius of the circle in which the particle rotates.

$$r = R\sin\theta$$

Acceleration of an object falling near the earth's surface, as measured from the earth frame, is F/m where F is the vector sum of the gravitational force and the centrifugal force.

So, $F = \frac{GMm}{R^2} - m\omega^2 R \sin\theta$

At the equator, $\theta = \pi/2$ and the centrifugal force is just opposite to the force of gravity.

Then, $F = mg - m\omega^2 R$

or $g' = g - \omega^2 R$

At pole, $\theta = 0^\circ$;

$F = mg$; $R\sin\theta = 0$

* So, g is minimum at the equator and maximum at the pole.

(c) Nonsphericity of the Earth

– The radius in the equatorial plane is about 21 km larger than the radius along the poles.

(d) Nonuniformity of the Earth

– The earth is not uniformly dense.

'Weighing' the Earth

$$g = \frac{GM}{R^2}$$

or, $$M = \frac{gR^2}{G}.$$

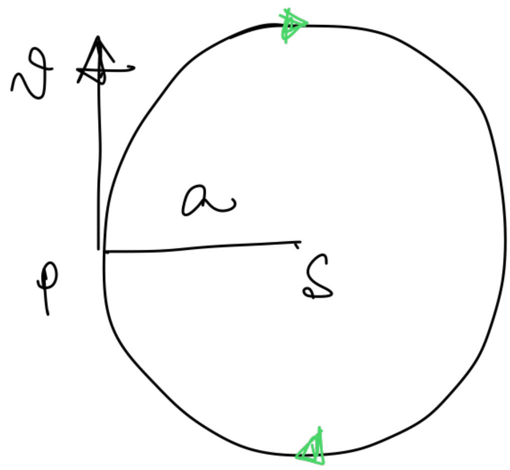
Now, $G = 6.67\times10^{-11}\ Nm^2/kg^2$

$g = 9.8\ m/s^2$

$R = 6400$ km

$M = 5.98\times10^{24}$ kg

Planets and Satellites



Mass of Sun M.

Mass of planet m.

Radius of the orbit, a.

Speed of the planet in the orbit be v.

So, $$\frac{GMm}{a^2} = m\left(\frac{v^2}{a}\right)$$

or, $\boxed{v = \sqrt{\frac{GM}{a}}}$ → Speed.

The speed of the planet is inversely proportional to the square root of the radius of the orbit.

Let T is the time taken in completing one revolution.

So, $T = \frac{2\pi a}{v}$

$$= \frac{2\pi a}{\sqrt{\frac{GM}{a}}} = \frac{2\pi}{\sqrt{GM}} a^{3/2}$$

or, $\boxed{T^2 = \frac{4\pi^2}{GM} a^3}$ → Time period.

The kinetic energy of the planet is

$K = \frac{1}{2} m v^2$

$$= \frac{1}{2} m . \frac{GM}{a} = \frac{GMm}{2a}$$

The gravitational potential energy of the Sun - planet system is,

$$U = -\frac{GMm}{a}$$

Thus, the total mechanical energy of the Sun - planet system is -

$$E = K + U = \frac{GMm}{2a} - \frac{GMm}{a}$$

$$\boxed{E = -\frac{GMm}{2a}} \rightarrow \text{Energy.}$$

* The total energy is negative. This is true for any bound system if the potential energy is taken to be zero at infinite separation.

* Once a satellite is placed in the desired orbit with the correct speed for that orbit, it will continue to move in that orbit under gravitational attraction of the earth.

All the equation derived above for planets are also true for satellite with representing the mass of earth and m, the mass of the satellite.

* The satellite that will appear to be stationary, are called Geostationary satellite. The time period of the satellite is 24 hours, same as earth.

$$T^2 = \frac{4\pi^2}{GM} a^3$$

putting values of $T = 24$ hr.

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

$M = 5.98 \times 10^{24}\ kg$

The radius of the geostationary orbit comes out to be, $a = 4.2 \times 10^4$ km.

The height above the surface of the earth is about 3.6×10^4 km.

$(R = 0.6 \times 10^4\ km \approx 6400\ km)$

[VII] Weightlessness in a Satellite

— A satellite moves round the earth in a circular orbit under the action of gravity.

The acceleration of the satellite is $\frac{GM}{R^2}$ towards the centre of earth.

Consider a body of m placed on a surface inside a satellite moving around the earth.

The forces on the body are —

(a) Gravitational pull of the earth, $\frac{GMm}{R^2}$

(b) The contact force N by the surface.

(c) Centrifugal force away from the centre of the earth.

So, $$\frac{GMm}{R^2} - N = m\left(\frac{v^2}{R}\right) = m\left(\frac{GM}{R^2}\right)$$

$$\text{or, } N = 0$$

i.e the surface doesn't exert any force on the body and hence its apparently weight is zero.

The feeling of weightlessness arises because one stays in a rotating frame.

(iii) Escape velocity

When a stone is thrown up it goes up to a max^m height and then returns. As the particle goes up, the gravitational potential energy increases and the kinetic energy of the particle decreases.
and the kinetic energy decreases.

Let initial velocity $= u$ and mass $= m$.

After height h, velocity becomes v.

By conservation of energy,

$$\frac{1}{2}mu^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 - \frac{GMm}{(R+h)}$$

$$\text{or, } \frac{1}{2}mv^2 = \left(\frac{1}{2}mu^2 - \frac{GMm}{R}\right) + \frac{GMm}{R+h}$$

The particle will reach max^m height when v is zero.

The particle will never return if

$$\frac{1}{2}mu^2 - \frac{GMm}{R} \geq 0 \quad \text{or, } \boxed{u \geq \sqrt{\frac{2GM}{R}}}$$

Putting the values of G, M and R the escape velocity from the earth is 11.6 Km/S.

Escape velocity for moon is 2.4 Km/S.

* The minimum energy needed to take particle infinitely away from the earth is called the binding energy of the earth- particle system.

The binding energy of the earth particle system is $\frac{GMm}{R}$

[M] Black holes

Consider a spherical body of mass M and radius R.

If the volume goes on decreasing while the mass remains the same, the escape velocity $\sqrt{\frac{2GM}{R}}$ from such a dense material will be very high. Suppose the radius is so small that,

$$\sqrt{\frac{2GM}{R}} \geq c \; ; c = 3\times10^{8} m/s \text{ is}$$

Speed of light.

According to the theory of relativity it is not possible to achieve a velocity greater than c for any material object.

Thus nothing can escape from such a dense material, not even the light. Such objects are known as Black Holes.

Inertial and Gravitational Mass.

- Let two objects of mass m_A and m_B. Equal forces F applied on each object

So, $F = m_A a_A$; $F = m_B a_B$

Thus $\frac{m_A}{m_B} = \frac{a_B}{a_A}$ or, $m_A = \frac{a_B}{a_A} m_B$

The mass so defined is called inertial mass.

- If F_A and F_B be the forces of attraction on the two objects due to the earth,

$$F_A = \frac{G m_A M}{R^2} \quad \text{and} \quad F_B = \frac{G m_B M}{R^2}$$

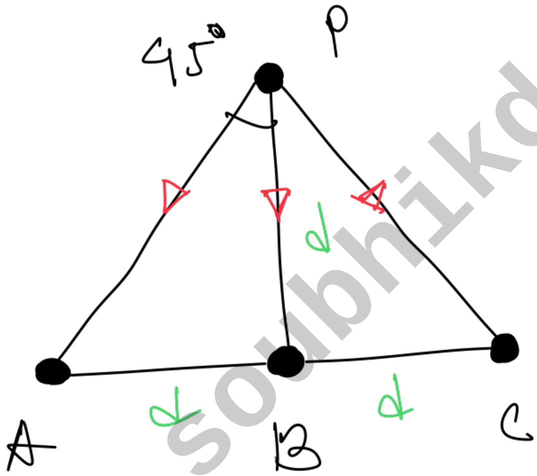
Then,

$$\frac{m_A}{m_B} = \frac{F_A}{F_B}$$

$$\text{or, } m_A = \frac{F_A}{F_B} m_B$$

The mass so defined is called gravitational mass.

Example



Mass of each particle is m. Find gravitational force on particle P.

— the force at P due to A = $F_A = \dfrac{Gm^2}{2d^2}$

$$F_B = \frac{Gm^2}{d^2}$$

$$F_C = \frac{Gm^2}{2d^2}$$

The resultant of F_A, F_B and F_C will be along PB. Clearly $\angle APB = \angle BPC = 45^\circ$

Taking the components of F_A and F_C along PB,

$F_A \cos 45^\circ$ and $F_C \cos 45^\circ$.

Hence the resultant of the three forces is —

$F_A \cos 45^\circ + F_B + F_C \cos 45^\circ$ along $\vec{PB}$