Newton's Laws of Motion

(1) First law of motion

- If the (vector) sum of all the forces acting on a particle is zero then and only then the particle remains unaccelerated (i.e. remains at rest or moves with constant velocity).

$$\boxed{\vec{a} = 0 \text{ if and only if } \vec{F} = 0}$$

- The concept of rest, motion or acceleration is meaningful only when a frame of reference is specified.

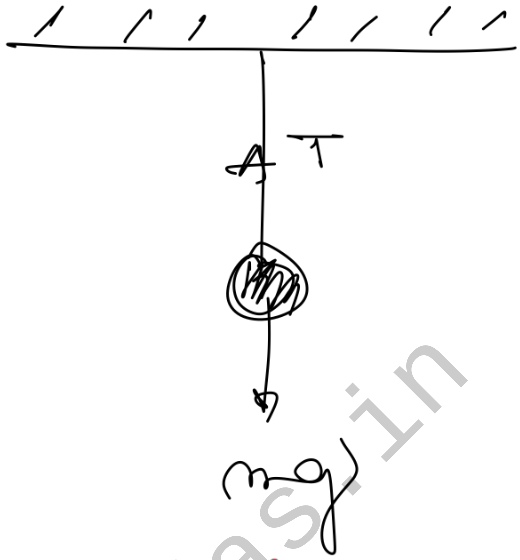
→ So, the validity of Newton's first law depends on the frame of reference from which the observer measures the state of rest, motion and acceleration of the particle. (Inertial frame).

- Newton's law not valid.
(Noninertial frame)

- All frames moving uniformly with respect to an inertial frame are themselves inertial.

* for the body at rest,

$T = mg$.



② Second law of Motion

- The acceleration of a particle as measured from an inertial frame is given by the (vector) sum of all the forces acting on the particle divided by its mass.

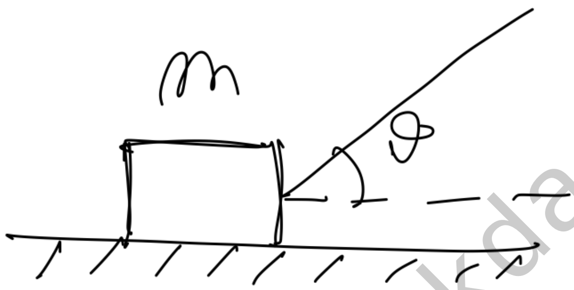
$\rightarrow \vec{a} = \vec{F}/m$ or, $\vec{F} = m.\vec{a}$

* Guidelines for Problems

- Decide the system

- Identify the forces
- Make a Free Body Diagram
- Choose axes and write equations.

Example:

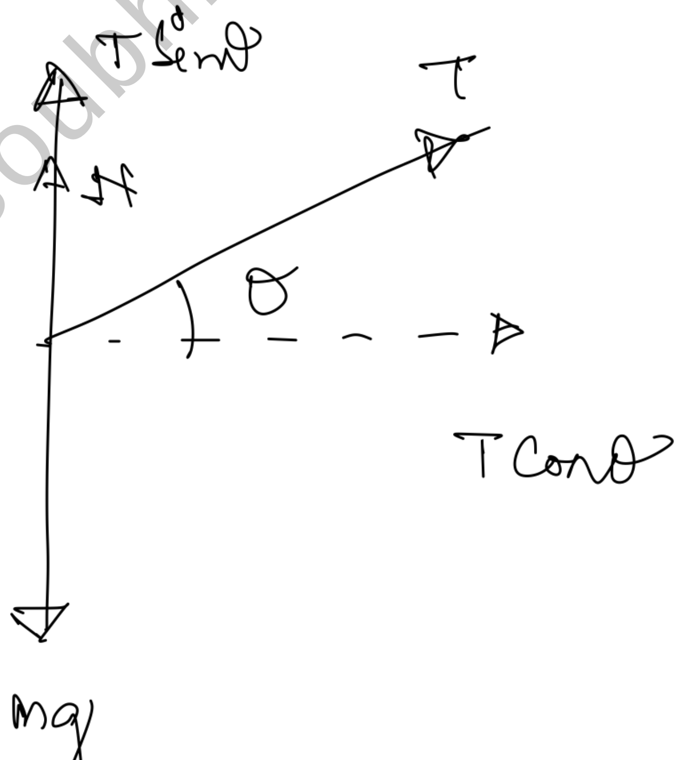


a = acceleration.

i. mg, ↓

ii. Contact force, N ↑

iii. Pull of the string T, along the string



x-axis,

$T\cos\theta = ma$ or, $T = \frac{ma}{\cos\theta}$

Y-axis,

$$N + T\sin\theta = mg$$

$$N = mg - ma\tan\theta$$

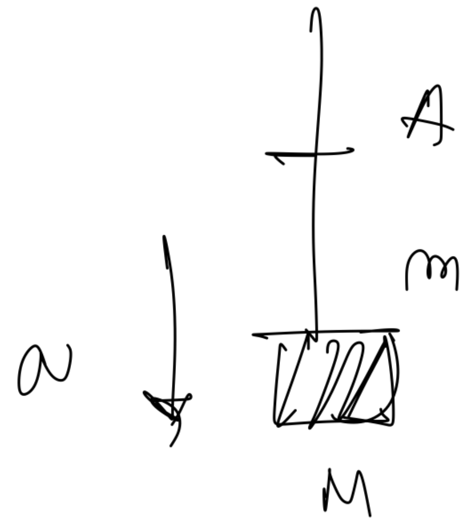
$$= m(g - a\tan\theta).$$

(3) Newton's third law

- If a body 'A' exerts a force $\vec{F}$ on another body B, then B exerts a force $-\vec{F}$ on A.

- The law is not strictly correct when interaction between two bodies separated by a large distance is considered.

※A



$(m+M)g \downarrow$

$T \uparrow$

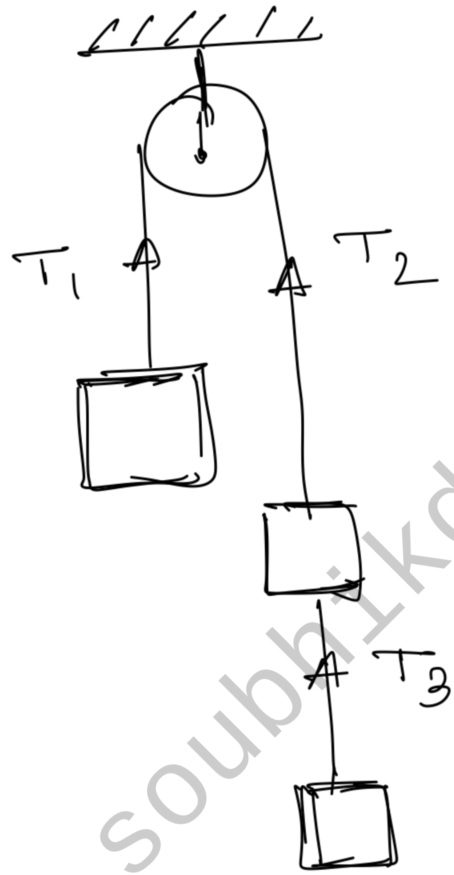
$(m+M)a$

For the system.

Downward force $= (M+m)g - T$

$= (M+m)a$.

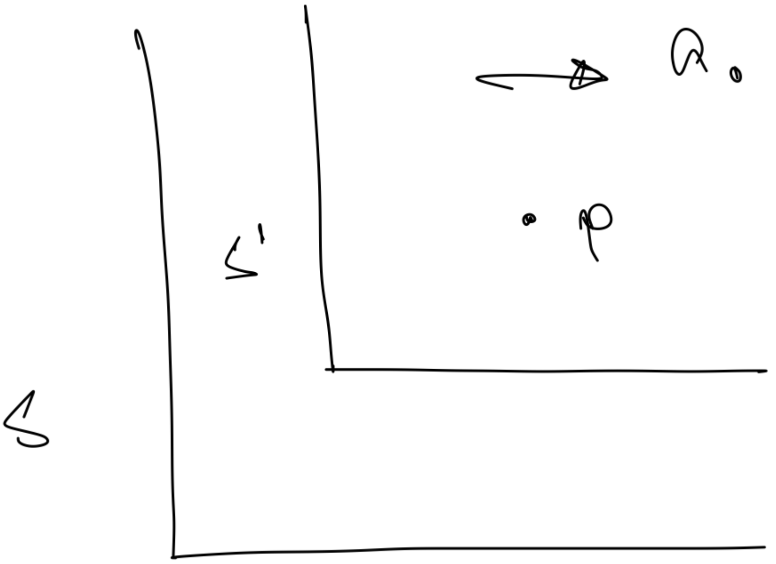
**



$T_1 = T_2$

Pseudo force

– To solve the motion of a body with respect to a non-inertial frame of reference.



- The frame of reference S' moves with a constant acceleration $\vec{a}_0$ with respect to an inertial frame S.
- The acceleration of the particle P with respect to S' is $\vec{a}_{P,S'}$.

As S' is translating with respect to S, we have,

$$\vec{a}_{P,S} = \vec{a}_{P,S'} + \vec{a}_{S,S'}$$

$$\text{or, } \vec{a}_{P,S'} = \vec{a}_{P,S} - \vec{a}_{S,S'}$$

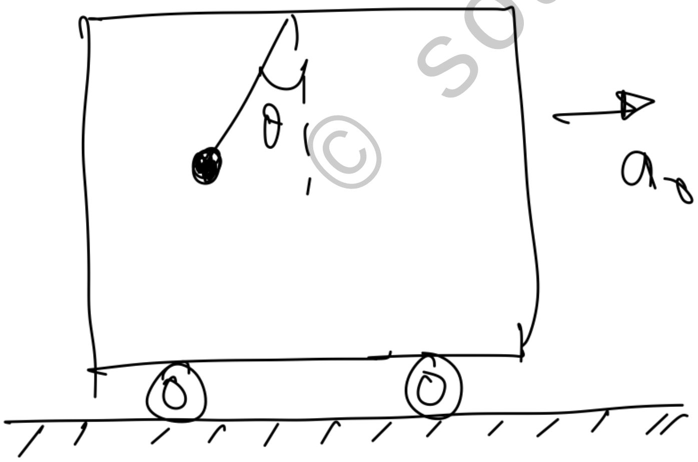
Now,

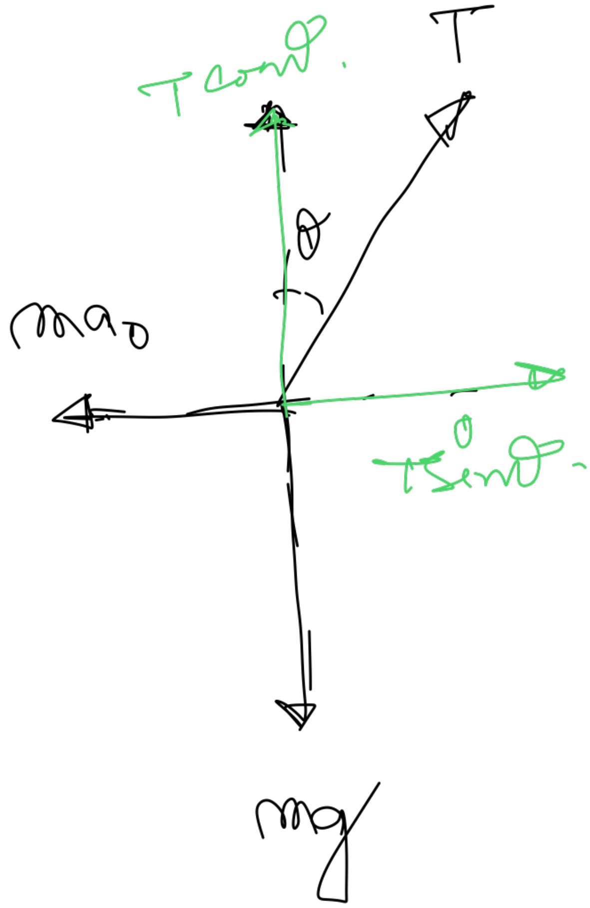
$$m\vec{a} = \vec{F} - m\vec{a}_0$$

or, $\vec{a} = \dfrac{\vec{F} - m\vec{a_0}}{m}$

- Such correction terms $-m\vec{a_0}$ in the list of forces are called pseudo force.
- The pseudo forces are also called inertial force although their need arises because of the use of noninertial frames.

* Example:





- T along the string
- mg, downward
- ma_0 towards left (pseudo force).

Now, $mg = T\cos\theta$

$ma_0 = T\sin\theta$

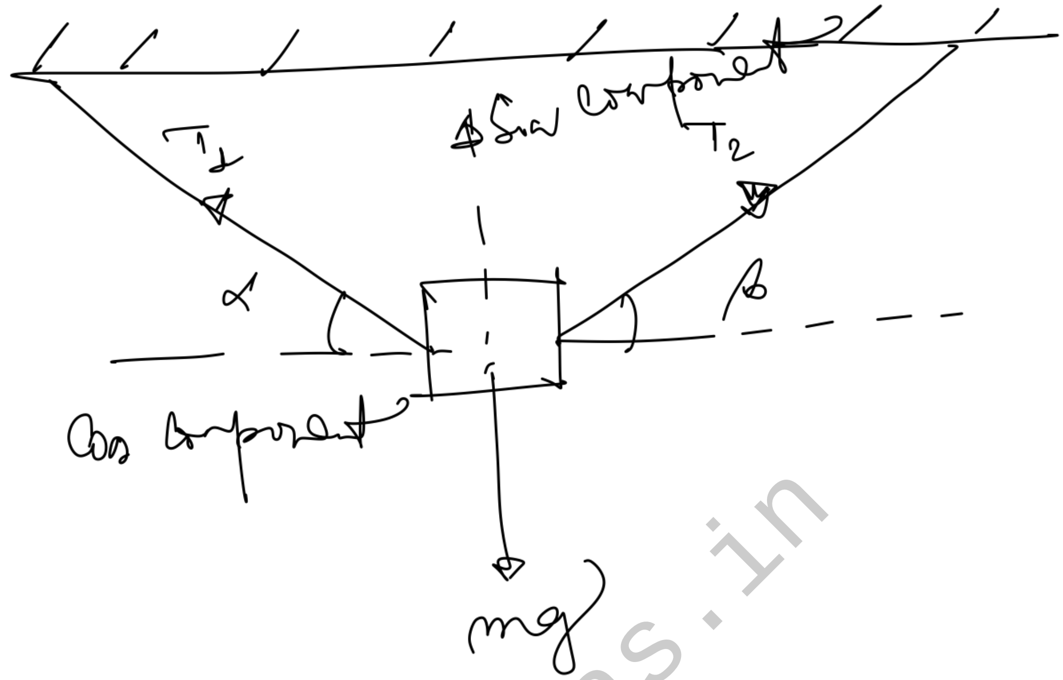
so, $\tan\theta = a_0/g$

$\Rightarrow \theta = \tan^{-1}(a_0/g)$

Inertia

– The unwillingness of a particle to change its state of rest or of uniform motion along a straight line is called as inertia.

Example 1



X-axis,

$$T_1 \cos\alpha - T_2 \cos\beta = 0$$

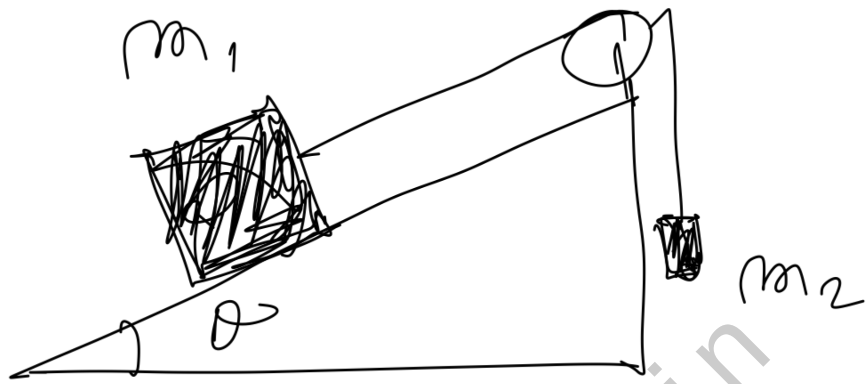
$$T_1 \cos\alpha = T_2 \cos\beta \quad \text{—— (1)}$$

Y-axis,

$$T_1 \sin\alpha + T_2 \sin\beta = mg \quad \text{—— (2)}$$

get T_1 & T_2 from eq^n ① & ②

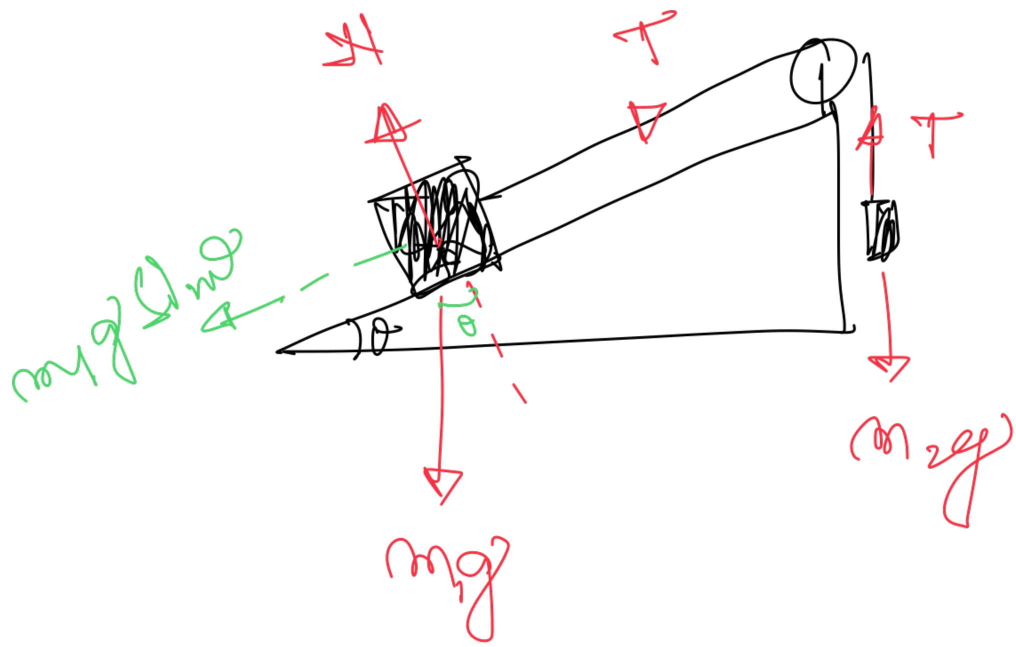
Example 2



– The system is at rest

(i) find the angle of the incline

(ii) force exerted by the incline on the body of mass m_1.



m_1 body.

component parallel to the incline,

$$T = m_1 g \sin\theta$$

m_2 body,

$$T = m_2 g$$

$$m_1 g \sin\theta = m_2 g$$

or, $\sin\theta = m_2/m_1$

$$\theta = \sin^{-1}(m_2/m_1)$$

taking components normal to the incline

$$N = m_1 g \cos\theta$$

$$= m_1 g \sqrt{1 - \left(\frac{m_2}{m_1}\right)^2}$$