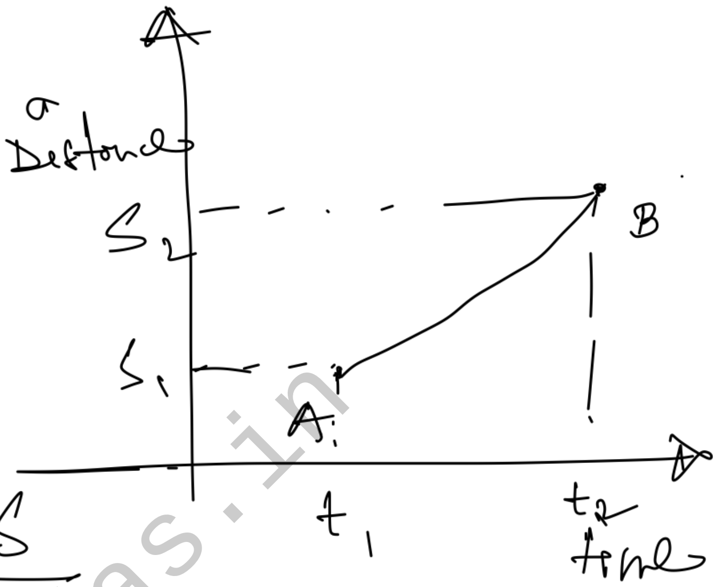
Rest & Motion

KINEMATICS

Average Speed = $\dfrac{\text{Distance Travelled}}{\text{time}}$



$$= \frac{S_2 - S_1}{t_2 - t_1} = \frac{\Delta S}{t_2 - t_1}$$

$$= \frac{\Delta S}{\Delta t}.$$

Instantaneous Speed =

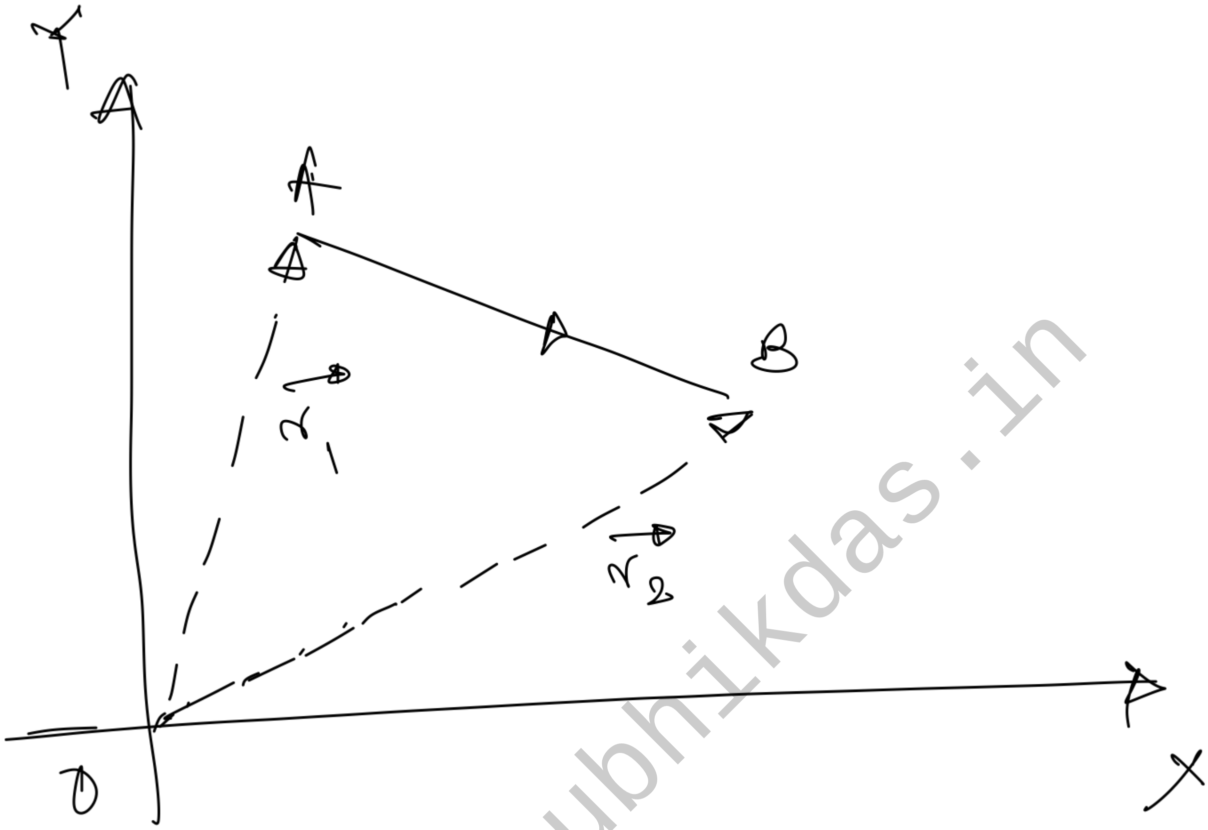
$$\lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = \frac{dS}{dt} = v$$

Slope at that particular instant point.

$$dS = v \cdot dt.$$

$\boxed{\text{Area under the curve of } v - t \text{ graph.}}$

- Distance — Scalar (only has magnitude)
- Displacement — Vector (both magnitude & direction)



$$\vec{v}_{av} = \frac{\overrightarrow{AB}}{t_2 - t_1}$$

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{r}_2 - \vec{r}_1$$

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$

- Instantaneous Velocity.

$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$; Δr is the displacement in time interval Δt.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}.$$

- Acceleration

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

average Acceleration.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right)$$

$$= \frac{d^2 s}{dt^2}$$

III Motion with constant Acceleration

Velocity at time $t=0$ is u

velocity at time $t=t$ is v.

Then Acceleration,

$$a = \frac{dv}{dt}$$

$$dv = a\,dt$$

$$\int_u^v dv = \int_0^t a.dt$$

$$\Rightarrow [v]_u^v = a[t]_0^t$$

$$\Rightarrow v-u = at$$

$$\Rightarrow \boxed{v = u+at}$$

Now, $v = \frac{dx}{dt}$ $\quad$ dx is the displacement.

$$v.dt = dx$$

$$\Rightarrow dx = (u+at).dt$$

$$\Rightarrow dx = u\,dt + at\,dt$$

$$\int_0^x dx = \int_0^t u\,dt + \int_0^t at\,dt$$

$$\Rightarrow x = u\int_0^t dt + a\int_0^t t\,dt$$

$$= u + a\left[\frac{t^2}{2}\right]_0^t$$

$$= u + \frac{at^2}{2}$$

So, $\boxed{x = u + \frac{1}{2}at^2}$

Now, $v = u + at$

$$v^2 = (u+at)^2$$

$$\boxed{v^2 = u^2 + 2ax}$$

Motion in a Straight Line.

Displacement at t^{th} Second =

$$S_t = u + \frac{a}{2}(2t-1) \quad \text{[Last 1 Second]}$$

VII Freely Falling Body.

$g \to 9.8\,m/s^2$ or $32\,ft/s^2$

(i) Vertically Downward ($g \to +ve$)

$$v = u + gt$$

$$s = ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gs$$

(ii) Vertically upward ($g \to -ve$)

$$v = u - gt$$

$$s = ut - \frac{1}{2}gt^2$$

$$v^2 = u^2 - 2gs$$

VIII Motion in a plane

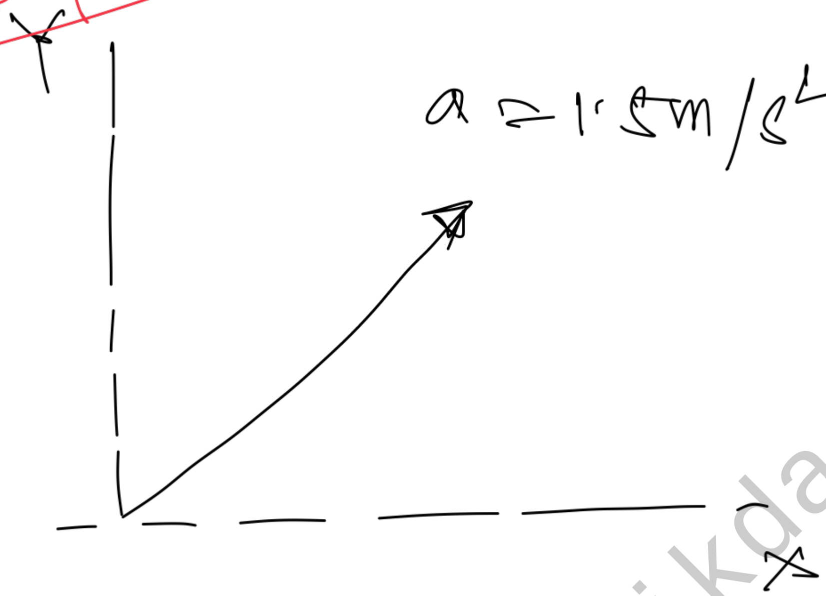
Break up into two independent problems of straight line motion, one along the x-axis and the other along y-axis.

$$\Delta \vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j}$$

$$\vec{v} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j}$$

$$\vec{a} = \frac{dv_x}{dt}\vec{i} + \frac{dv_y}{dt}\vec{j}$$

Example

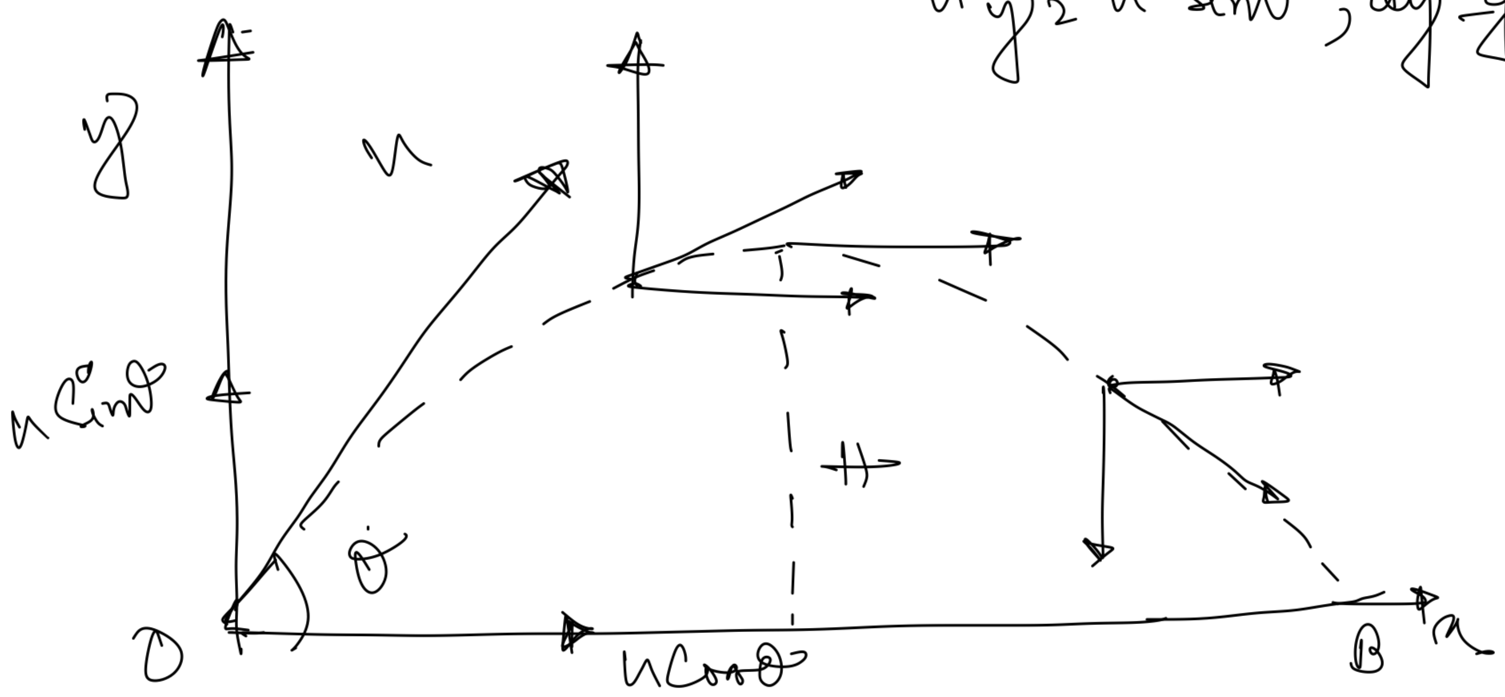


* find the X & Y component of Acceleration, velocity & Displacement.

Projectile Motion

$u_x = u\cos\theta \,;\, a_x = 0$

$u_y = u\sin\theta \,;\, a_y = g$



Horizontal Motion

$$x = u_x t + \frac{1}{2} a_x t^2 \quad (a_x = 0)$$

$$= ut \cos\theta$$

As indicated, the x-component of the velocity remains constant as the particle moves.

Vertical Motion

$$v_y = u_y - gt$$

$$y = u_y t - \frac{1}{2} g t^2$$

$$v_y^2 = u_y^2 - 2gy$$

Time of flight

Time taken t for OB distance.

$$OB = u_x t + \frac{1}{2} a_x t^2 \quad (a_x = 0)$$

$$= ut \cos\theta.$$

At point B, $v_y = 0$

Now, $v_y = u_y t - \frac{1}{2} a_y t^2$

$0 = ut\sin\theta - \frac{1}{2} g t^2$

$\Rightarrow t\left(u\sin\theta - \frac{1}{2} g t\right) = 0$

So, $t = 0$ or $\boxed{t = \frac{2u\sin\theta}{g}}$

Time of Flight.

Range

$OB = (u\cos\theta) t$

$= u\cos\theta \cdot \frac{2u\sin\theta}{g}$

$$\boxed{OB = \frac{u^2 \sin 2\theta}{g}}$$

Maximum Height Reached

$$v_y = u_y - gt$$

$$= u\sin\theta - gt$$

at max^m Height, $v_y = 0$

so, $u\sin\theta = gt$

$$t = \frac{u\sin\theta}{g}$$

Max^m Height, $H = u_y t - \frac{1}{2} a_y t^2$

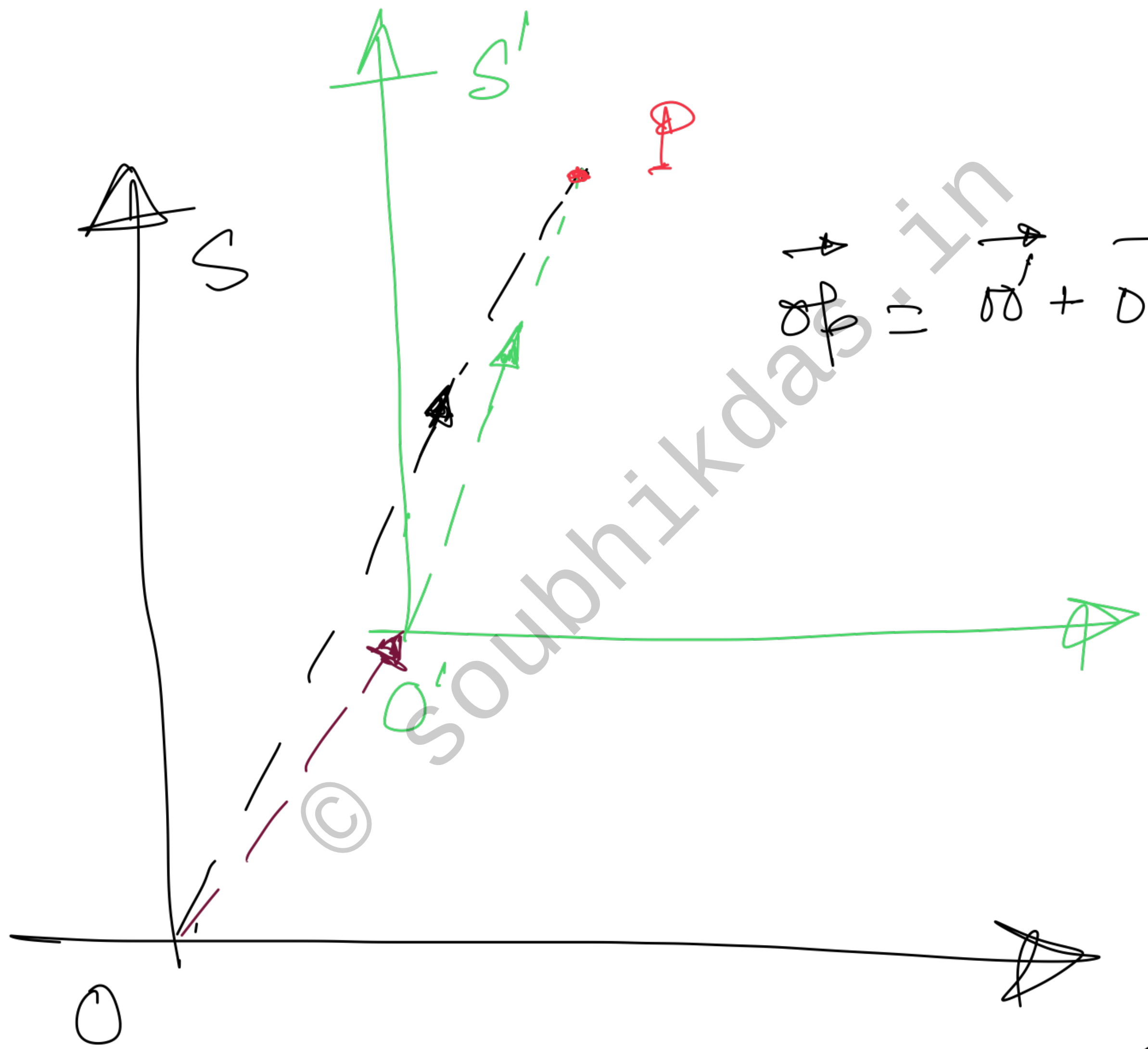
$$= u\sin\theta \cdot \left(\frac{u\sin\theta}{g}\right) - \frac{1}{2} g \left(\frac{u\sin\theta}{g}\right)^2$$

$$= \frac{u^2\sin^2\theta}{g} - \frac{u^2\sin^2\theta}{2g}$$

$$\boxed{H = \frac{u^2\sin^2\theta}{2g}}$$

(VIII) Change of frame

- A frame can be chosen according to a given frame of reference. (according to the convenience of the problem)



$$\vec{OP} = \vec{OO'} + \vec{O'P}$$

- The position of the frame S' (origin) with respect to S is $\vec{OO'} = \vec{r}_{S',S}$

Position vector OP (S plane) $= \vec{OP} = \vec{r}_{P,S}$

position vector $O'P$ (S' plane) $= \vec{r_P} = \vec{r}_{P,S'}$

So, $\vec{OP} = \vec{OO'} + \vec{O'P}$

$$\vec{r}_{P,S} = \vec{r}_{S',S} + \vec{r}_{P,S'}$$

So, for velocity -

$$\frac{d}{dt}\left(\vec{r}_{P,S}\right) = \frac{d}{dt}\left(\vec{r}_{S',S}\right) + \frac{d}{dt}\left(\vec{r}_{P,S'}\right)$$

$v_{S',S} \rightarrow$ Velocity of the frame S' with respect to S.

$$\vec{v}_{P,S} = \vec{v}_{S',S} + \vec{v}_{P,S'}$$

* $\frac{d}{dt}$ has the same meaning in both the frames.

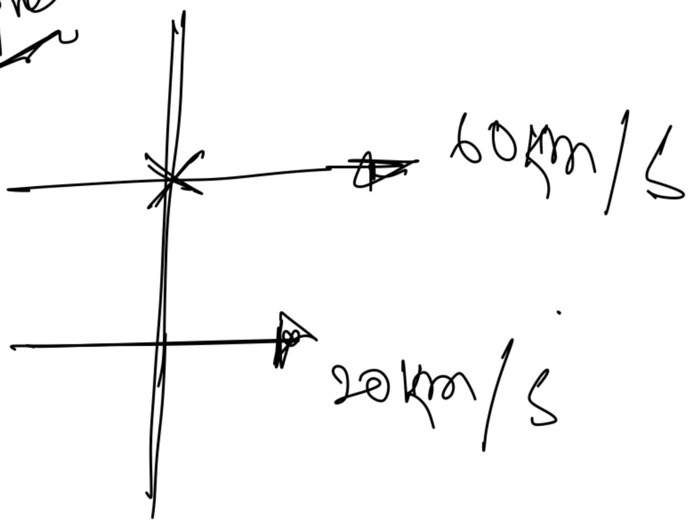
* The assumptions are not correct if the velocity of one frame w.r.t the other is so large that it is comparable to 3×10^8 m/s or if it rotates w.r.t the other.

So, it can be written,

$$\vec{v}_{P,S'} = \vec{v}_{P,S} - \vec{v}_{S',S}$$

w.r.t the moving plane.

Example



Bullet → $\vec{v}_{B,T}$ w.r.t Train

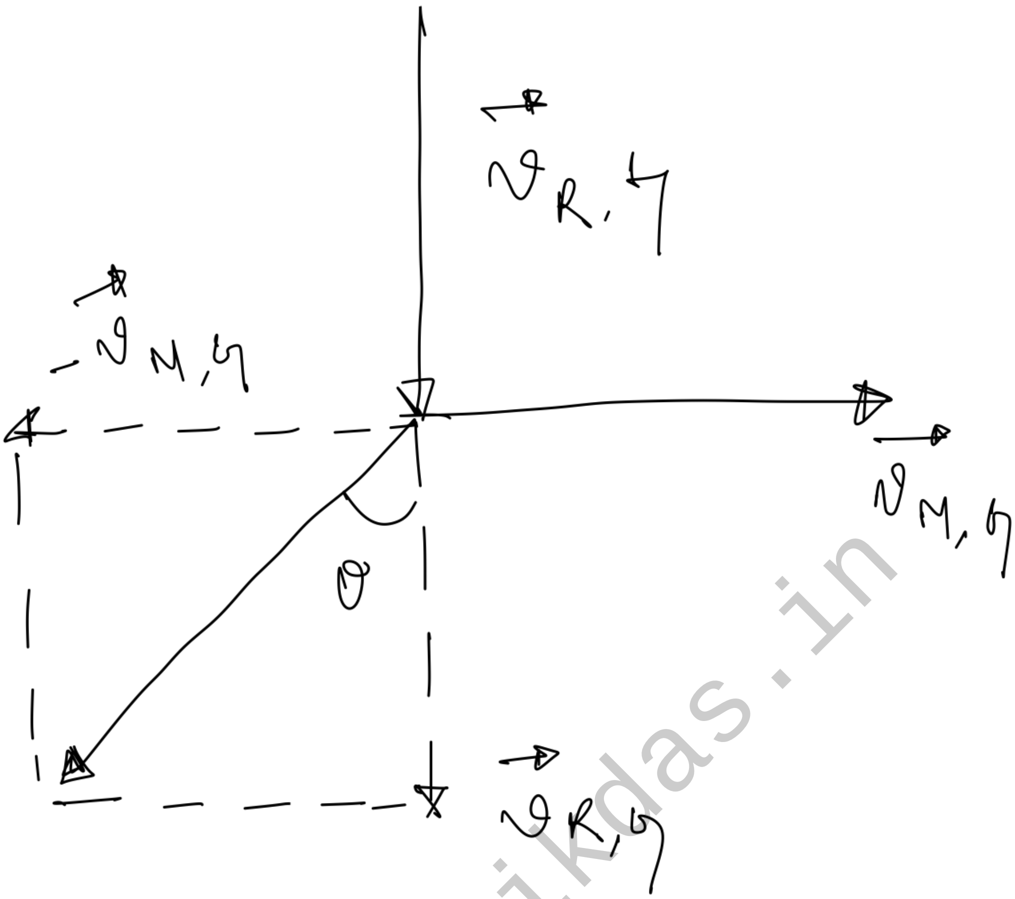
Train → $\vec{v}_{T,G}$ w.r.t ground

$$\vec{v}_{B,G} = \vec{v}_{B,T} + \vec{v}_{T,G}$$

$= 60 + 20$

$= 80$ km/s.

Example: Rain



$$v_{R,M} = \vec{v}_{R,G} - \vec{v}_{M,G}$$

$$-\vec{a}_{P,S'} = \vec{a}_{P,S} - \vec{a}_{S',S}$$ → Same frame

or, $$\vec{a}_{P,S} = \vec{a}_{P,S'} + \vec{a}_{S',S}$$ → different frame.

- If S' moves with respect to S at a uniform velocity.

$$\vec{a}_{S',S} = 0$$

So, $\vec{a}_{P,S} = \vec{a}_{P,S'}$

i.e if two frames are moving with respect to each other with uniform velocity, acceleration of a body is same in both the frames.