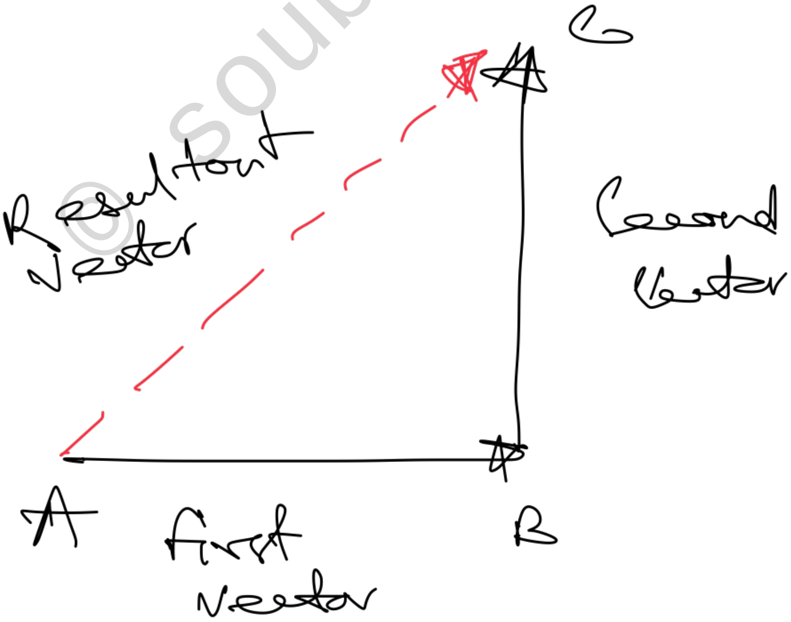
Vectors & Scalar

- Certain physical quantities are completely described by a numerical value alone (with units specified) and are added according to the ordinary rules of algebra. Such quantities are called **Scalars**.

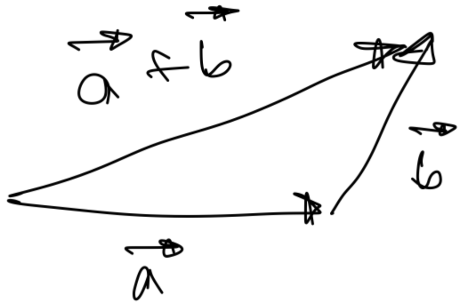
- The physical quantities which have magnitude and direction and which can be added according to the triangle rule, are called **Vector quantities**.



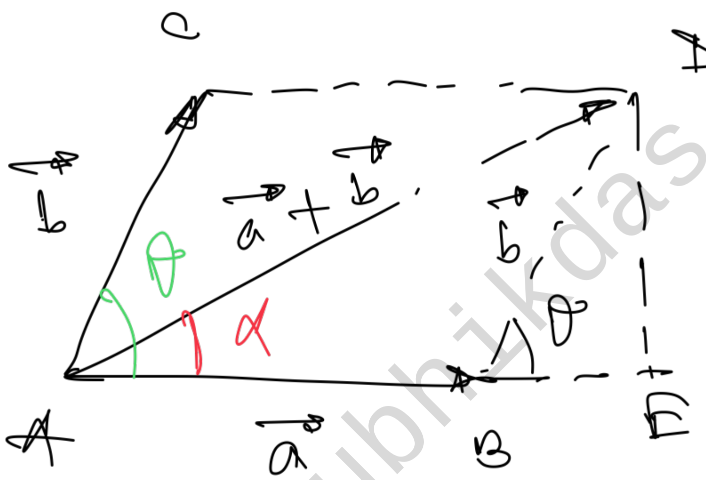
⇒ Triangle rule of Addition.

* Two vectors are called equal if their magnitude and directions are same.

Addition of Vector



- The tail of $\vec{b}$ coincides with the head of $\vec{a}$.
- The vector joining the tail of $\vec{a}$ with the head of $\vec{b}$ is the vector sum of $\vec{a}$ and $\vec{b}$.



- The diagonal through the common tail.

Suppose, $|\vec{a}| = a$ & $|\vec{b}| = b$

The angle between $\vec{a}$ & $\vec{b}$ is θ.

Now, $AD^2 = AE^2 + DE^2$

$$= (a + a\cos\theta)^2 + (b\sin\theta)^2$$

$$= a^2 + 2ab\cos\theta + b^2$$

Then, the magnitude of $\vec{a} + \vec{b}$ is,

$$\boxed{AD = \sqrt{a^2 + b^2 + 2ab\cos\theta}}$$

- If angle with $\vec{a}$ is α, then

$$\boxed{\tan\alpha = \frac{DB}{AB} = \frac{b\sin\theta}{a + b\sin\theta}}$$

III Multiplication of a vector by number

$\vec{b} = K\vec{a}$ — K is a number.

- If K is positive, the direction of $\vec{b}$ is same as $\vec{a}$.

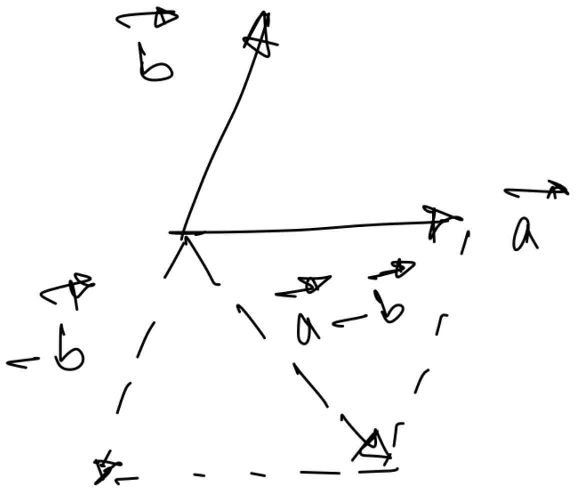
- If K is negative, the direction of $\vec{b}$ is opposite to $\vec{a}$.

* $\vec{a} = a.\vec{u}$ $\quad |\vec{a}| = a$

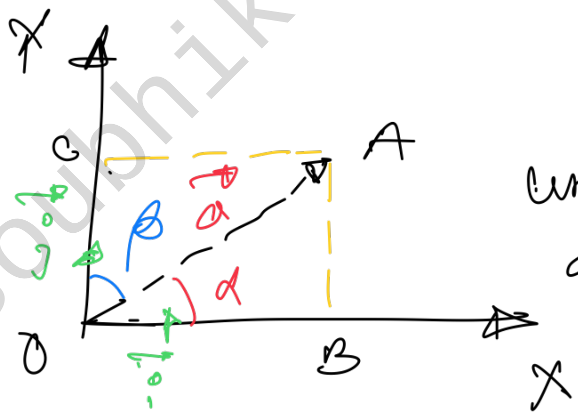
$\vec{u}$ is a unit vector of unit magnitude in the direction of $\vec{a}$.

Subtraction of Vector

— To subtract $\vec{b}$ from $\vec{a}$, invert the direction of $\vec{b}$ and add to $\vec{a}$



Resolution of Vector



$\vec{i}$, $\vec{j}$ denotes vector of unit magnitude along OX and OY.

(2 - dimensional)

$$\vec{a} = \vec{OA} = \vec{OB} + \vec{OC}$$

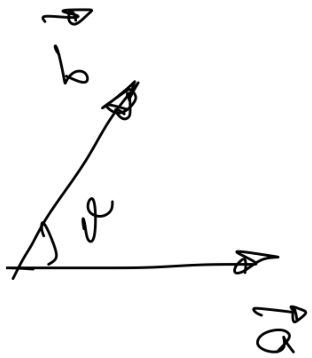
$$= a\cos\alpha\,\vec{i} + a\sin\alpha\,\vec{j}$$

$$= a\cos\alpha\,\vec{i} + a\sin(90^\circ-\beta)\,\vec{j}$$

$$= a\cos\alpha\,\vec{i} + a\cos\beta\,\vec{j}$$

Dot Product or Scalar Product of Two Vectors

$$\boxed{\vec{a}\cdot\vec{b} = ab\cos\theta}$$



For two mutually perpendicular vectors the dot product is zero cause $\cos 90^\circ = 0$

- Dot product is commutative and distributive.

$$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$$

$$\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$$

If $\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$

$\vec{b} = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$

Since, $\vec{i}, \vec{j}$ & $\vec{k}$ are mutually orthogonal

$$\vec{i}\cdot\vec{j} = \vec{i}\cdot\vec{k} = \vec{j}\cdot\vec{i} = \vec{j}\cdot\vec{k} = \vec{k}\cdot\vec{i} = \vec{k}\cdot\vec{j} = 0$$

also, $\vec{i}\cdot\vec{i} = 1.1.\cos 0 = 1$

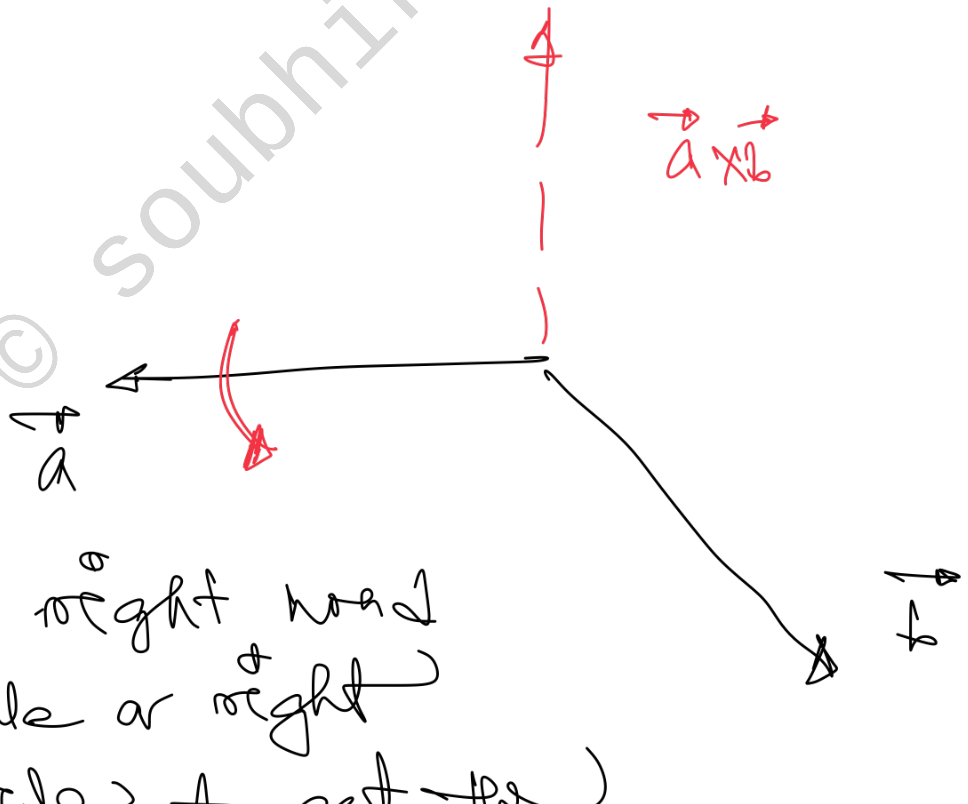
$\vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k} = 1$

$$\boxed{\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z}$$

III) Cross Product or Vector Product

$$\boxed{|\vec{a}\times\vec{b}| = ab\sin\theta}$$

$\vec{a}\times\vec{b}$ is itself a vector.



- Apply right hand thumb rule or right hand rule to get the direction of $\vec{a}\times\vec{b}$

- Non commutative

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

- Distributive

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

- Doesn't follow Associative law

$$\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$$

Since,

$$\left.\begin{array}{l} \vec{i} \times \vec{j} = \vec{k} \\ \vec{j} \times \vec{k} = \vec{i} \\ \vec{k} \times \vec{i} = \vec{j} \end{array}\right\}$$

&,

$$\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0 \quad [0°, 180°]$$

Let,
$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$
$$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Very careful

$$= (a_y b_z - a_z b_y)\hat{i} - (a_x b_z - a_z b_x)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$$